Current-Induced Motion of Narrow Domain Walls and Dissipation in Ferromagnetic Metals

M. Benakli,* J. Hohlfeld,† and A. Rebei‡
Seagate Research Center, Pittsburgh, Pennsylvania 15222, USA
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Spin transport equations in a non-homogeneous ferromagnet are derived in the limit where the sd exchange coupling between the electrons in the conduction band and those in the d band is dominant. It is shown that spin diffusion in ferromagnets assumes a tensor form. The diagonal terms are renormalized with respect to that in normal metals and enhances the dissipation in the magnetic system while the off-diagonal terms renormalize the precessional frequency of the conduction electrons and enhances the non-adiabatic spin torque. To demonstrate the new physics in our theory, we show that self-consistent solutions of the spin diffusion equations and the Landau-Lifshitz equations in the presence of a current lead to a an increase in the terminal velocity of a domain wall which becomes strongly dependent on its width. We also provide a simplified equation that predicts damping due to the conduction electrons.

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Dynamics of magnetic domain walls (DW) is a classic topic [1, 2, 3] that recently received a lot of attention due to new fabrication and characterization techniques that permit their study at the nanometer scale. Moreover, the subject of spin dynamics in the presence of large inhomogeneities is currently of great interest experimentally and theoretically due to the potential applications in various nano-devices, especially magnetic storage [4]. One particular area that is still not well understood is the interaction of domain walls (DWs) with polarized currents. The question here is how best to represent the contribution of the spin torque to the dynamics of the magnetization [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] . So far attention has been focused on wide DWs where it was shown that terminal velocities are independent of the DW width [9, 14].

This paper extends previous treatments to the case of thin, less than 100 nm, DWs. One of the main objectives of our work is to expose the interplay between linear momentum relaxation and spin relaxation as the conduction electrons traverse a thin DW. This interplay originates from the strong exchange interaction between the conduction s electrons and the localized d moments, and makes the terminal velocities as well as the transport parameters of the conduction electrons dependent on the configuration of the local magnetization. This leads to an enhancement of the non-adiabatic contribution of the spin torque to the DW motion and opens the way to study spin torque-induced magnetization dynamics in thin DWs in greater depth by measurement of DW velocities. Moreover, we show that the interaction of the conduction electrons and the d moments is also relevant for homogeneously magnetized metallic systems, where it is at the origin of intrinsic damping. Our work can be easily adapted to magnetic multilayer structures and hence the equations derived here are capable to treat non-collinear magnetization geometries as opposed to that in ref. [16] which deal only with collinear configurations. Narrow DWs can exist either naturally [17, 18] or artificially [19, 20] and we hope the results discussed here show the potential benefits of studying dissipation in DW-like structures.

To derive the spin coupling of the s electrons to the magnetization, we adopt the sd picture which has been the basis for most of the studies in DW motion [9]. In the following we use (l, m, n) for moment indexes, and (i, j, k) for space indexes. In addition the transverse domain wall is assumed to extend in the x direction, with magnetization pointing in the z direction. We start from the Boltzmann equation satisfied by the 2×2 distribution function of the conduction electrons, $\mathbf{f} = f^e + \mathbf{f^s} \cdot \sigma$, where σ_l $(l = 1, 2, 3)$ are Pauli matrices, in the presence of the magnetization $\mathbf{M}$ of the system and an external electric field $\mathbf{E}$:

$$\begin{aligned} \partial_t \mathbf{f} + \mathbf{v} \cdot \nabla \mathbf{f} \quad & + e\left(\mathbf{E} + \mathbf{v} \times \mathbf{H}\right) \cdot \nabla_p \mathbf{f} + \\ i\left[\mu_B \sigma \cdot \mathbf{H}_{sd}, \mathbf{f}\right] \quad & = -\frac{f^e - f_0^e}{\tau_p} - \frac{\mathbf{f} - \mathbf{f}_0^s}{\tau_{sf}}. \end{aligned} \tag{1}$$

The sd exchange field is $\mathbf{H}_{sd}(\mathbf{x}, t) = J\,\mathbf{M}(\mathbf{x}, t)/\mu_B$ with $J \approx 1.0$ eV, and τ_p, τ_{sf} are the momentum and spin relaxation times, respectively [15, 21, 22]. The variables $\mathbf{v}$, e, and μ_B are the velocity, the charge and the magnetic moment of the s electrons, respectively. f_e^0 and $\mathbf{f}_s^0$ are the equilibrium charge and spin distribution.

The conduction electrons have a polarization $\mathbf{m} = \mu_B \int \frac{d\mathbf{p}}{(2\pi)^3} Tr \sigma \mathbf{f}$ and carry a charge current $\mathbf{j}_c = e \int \frac{d\mathbf{p}}{(2\pi)^3} \mathbf{v} Tr \mathbf{f}$, as well as a spin current

$$\mathbf{j}_s = \int \frac{d\mathbf{p}}{(2\pi)^3} \mathbf{v} Tr \sigma \mathbf{f}. \tag{2}$$

In the following we use normalized definitions of the moments, i.e. $||\mathbf{M}|| = ||\mathbf{m}|| = 1$. The d electrons will be assumed to satisfy a Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{d\mathbf{M}}{dt} = -\mathbf{M}\times\left(\gamma\mathbf{H}_{eff}+\frac{1}{\tau_{ex}}\mathbf{m}\right)+\alpha_{\mathrm{pd}}\mathbf{M}\times\frac{d\mathbf{M}}{dt}, \tag{3}$$

where τ_{ex} is the inverse of the precessional frequency, $\omega_c = J/\hbar$, of the conduction electrons due to the exchange field. $\mathbf{H}_{eff}$ is the total field acting on the magnetization which includes the exchange field between the d-moments, the demagnetization field and the anisotropy field. In metals, the main source of dissipation is believed to be due to the conduction electrons which in our theory is accounted for explicitly within the limitations of the sd model [23]. Hence, the damping constant α_{pd} is assumed to be due to dissipation caused by channels other than the conduction electrons such as phonons or defects.

In inhomogeneous magnetic media, the sd exchange term becomes comparable to that of the Weiss molecular field and hence the effect of the conduction electrons on the magnetization should be taken beyond the linear response approach. Going beyond the linear theory will allow us to see how the presence of the background magnetization affects the transport properties of the conduction electrons. We believe this is especially true in transition metal nano-magnetic devices where the hybridization of the s and d electrons is strong. Using standard many-body methods [15], the diffusion contribution to the spin current can be found

$$j_s^{li}(t,\mathbf{x}) = -\mathcal{D}^{ln}(t,\mathbf{x})\,\nabla_i m_n(t,\mathbf{x}), \tag{4}$$

where $\mathcal{D}$ is a diffusion tensor with effective relaxation time τ which will be assumed equal to the momentum relaxation ($\tau \approx \tau_p$). The $\mathcal{D}$ tensor obeys the reduced symmetry of the ferromagnetic state and is [15]

$$\mathcal{D} = D_\perp\begin{bmatrix} 1+\tau^2\Omega_x^2 & \tau\Omega_z+\tau^2\Omega_x\Omega_y & -\tau\Omega_y+\tau^2\Omega_z\Omega_x \\ -\tau\Omega_z+\tau^2\Omega_y\Omega_x & 1+\tau^2\Omega_y^2 & \tau\Omega_x+\tau^2\Omega_y\Omega_z \\ \tau\Omega_y+\tau^2\Omega_z\Omega_x & -\tau\Omega_x+\tau^2\Omega_y\Omega_z & 1+\tau^2\Omega_z^2 \end{bmatrix}, \tag{5}$$

where $\Omega = J\mathbf{M}/\hbar$, $D_\perp = \frac{D_0}{1+(\tau\omega_c)^2}$ with $D_0 = \frac{1}{3}\mathrm{v_f}^2\tau_p$ being the diffusion constant of the electron gas with Fermi velocity $\mathrm{v_f}$. It should be observed that in the presence of spin-orbit coupling, the symmetry of the diffusion tensor will be the same as given here but the separation of the relaxation times in independent channels of momentum and spin relaxation will not be valid. In the following, the effect of the electric field is taken only to first order.

The symmetry of the spin current is best revealed by going to a local frame where the magnetization lies in the z-direction. In this frame, one obtains for $\mathbf{E}=\mathbf{0}$

$$j_\perp = -D_{\mathrm{eff}}\frac{dm}{dx},\;\; j_z = -D_0\frac{dm_z}{dx}, \tag{6}$$

where $m(x) = m_x(x) - im_y(x)$, and $D_{\mathrm{eff}} = D_\perp + iD_{xy}$ is an effective diffusion coefficient with $D_{xy} = D_\perp\tau\omega_c$. From the divergence of the spin current we get the steady-state equation for the spin accumulation,

$$\frac{d^2m}{dx^2} = \frac{m}{\lambda_{\mathrm{eff}}^2},\;\; \frac{d^2m_z}{dx^2} = \frac{m_z-m_0}{\lambda_{sdl}^2}, \tag{7}$$

where $\lambda_{\mathrm{eff}}^2 = \tau_{\mathrm{eff}}D_{\mathrm{eff}}$ with $\tau_{\mathrm{eff}} = 1/(\frac{1}{\tau_{sf}} - i\omega_c)$, m_0 is the equilibrium spin density, and λ_{sdl} is the longitudinal spin diffusion length typically in the range of 5-100 nm. The general solutions for the complex accumulation are of the form $m(x) = A\exp[-x/\lambda_{\mathrm{eff}}] + B\exp[x/\lambda_{\mathrm{eff}}]$, i.e. they show an exponential decrease (or increase) and oscillations from a local inhomogeneity in $\mathbf{M}$. In the limit of a large sd exchange field the period of the oscillations is $\frac{\mathrm{v_f}}{\omega_c}$ which corresponds to the coherence length $1/|k^\uparrow - k^\downarrow|$ in the ballistic approach, where $k^\uparrow$ is the spin-up momentum.

Our expressions for the spin current generalize those used currently in the literature [9]. We find that the diffusion constant D_0 is now renormalized by $1/(1+(\tau\omega_c)^2)$ which means that precession in the exchange field reduces diffusion. Moreover, the precession gives rise to off-diagonal terms in the diffusion tensor which reflect the local 2D rotational symmetry around $\mathbf{M}$.

The origin of the off-diagonal term $\mathcal{D}_{xy}$ can be understood qualitatively in terms of flux. First we rewrite it in the following form

$$D_{xy} = (\frac{1}{3}\mathrm{v_f}^2\tau_p)\frac{\tau_p\omega_c}{1+(\tau_p\omega_c)^2} = \frac{1}{3}\frac{\mathrm{v_f}^2\omega_c}{\nu^2+\omega_c^2} \tag{8}$$

where $\nu = \frac{1}{\tau_p}$. In the limit of fast precession, $\nu \ll \omega_c$, we have $D_{xy} = \frac{1}{3}{\rm v_f}^2/\omega_c$. Next if we set ${\rm v_f}/\omega_c = L_m$, then L_m is the distance a spin typically goes before it 'converts' into the spin at 90-degrees to that which it started with. The corresponding contribution to the flux has an obvious interpretation - the source of spin x, m_x, is particles coming from a distance L_m away where they had spin y, m_y. The flux can be derived from a simple 'kinetic' argument. A distance L_m upstream, the density is $m_y = m_y^0 + L_m \frac{dm_y}{dx}$ and a distance L_m downstream, $m_y = m_y^0 - L_m \frac{dm_y}{dx}$. The flux of particles with spin x, m_x, crossing a point, coming from upstream, is ${m_y}^{\uparrow}{\rm v_f}/3$ and from downstream it is ${m_y}^{\downarrow}{\rm v_f}/3$. The difference is then $({m_y}^{\uparrow} - {m_y}^{\downarrow}){\rm v_f}/3 = 2{\rm L_m}\frac{\rm dm_y}{\rm dx}{\rm v_f}/3$ which, within a factor of 2, is our off-diagonal flux. In short, the off-diagonal terms are the corrections induced by precession on the diffusion process.

To get the effective equation for $\mathbf{M}$, we use equation 3 to express $\mathbf{m}$ in terms of the magnetization $\mathbf{M}$, then use the implicit solution back into the equation for $\mathbf{m}$, Eq. 1. We find that the equation of motion for the magnetization becomes

$$\begin{aligned}\beta\frac{d\mathbf{M}}{dt} &= -\gamma\mathbf{M}\times\mathbf{H}_{eff} + \mathbf{a}\cdot\nabla\mathbf{M} + (\alpha_{\rm pd} - \xi)\,\mathbf{M}\times\frac{d\mathbf{M}}{dt} \\ &\quad -\xi\gamma\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{eff}) + \nabla\cdot\mathcal{D}\nabla\mathbf{m}, \end{aligned} \tag{9}$$

where $\mathbf{a} = P(\mu_B/e)Tr\mathbf{j}$, $\beta = 1 + m_0 + \alpha_{\rm pd}\xi$, (or MB$\beta' = (1 + m_0 + \xi^2)$), $\xi = \tau_{ex}/\tau_{sf}$ is the ratio of the precessional time to the spin relaxation time of the conduction electrons, and P is the spin current polarization [9]. The second term on the right is the adiabatic spin torque while the last term is the diffusion contribution. The third and the fourth terms are equivalent to the non-adiabatic spin torque with the original damping $\alpha_{\rm pd}$ from Eq. 3 added to the third term. For uniform magnetization, $\nabla\mathbf{M} = \nabla\mathbf{m} = 0$, and damping constant $\alpha_{\rm pd} = 0$, Eq. 9 reduces to

$$\begin{aligned}\frac{d\mathbf{M}}{dt} &= -\gamma\frac{\xi^2+\beta}{\xi^2+\beta^2}\mathbf{M}\times\mathbf{H}_{\rm eff} \\ &\quad -\gamma\xi\frac{m_0}{\beta^2+\xi^2}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{\rm eff})\,. \end{aligned} \tag{10}$$

Hence, we are able to predict damping due to conduction electrons, quantify the corresponding damping constant $\alpha_{el} = \gamma\xi m_0/(\beta^2 + \xi^2)$, and identify its origin as the spin torque contribution of the conduction electrons. We have written Eq. 10 in the LL form, but it equally can be written in the LLG form. We have already shown in ref.[15] that the magnetization dynamics of a thin film embedded between two normal conductors and subjected to an electric field is not well described by closed LL (or LLG) equations.

Next we discuss qualitatively the effect of the diagonal and off-diagonal terms of the diffusion tensor on the velocity of a domain wall, of width λ. If we ignore the spatial dependence of the diffusion tensor elements and replace the Laplacian in the diffusion equation by $1/\lambda^2$, then we recover equations similar to those discussed by Zhang and Li [9] but with renormalized spin flip scattering rate, $1/\tau_{sf} \to 1/\tau_{sf}^N = 1/\tau_{sf} + D_0/\lambda^2$, and renormalized precessional frequency, $1/\tau_{ex} \to 1/\tau_{ex}^N = 1/\tau_{ex} - D_{xy}/\lambda^2$. Therefore, the velocity and the effective damping of the DW dependent on the size of the inhomogeneities in the magnetization. This can be understood qualitatively from the results in [9] which showed that the DW velocity v for a wide DW, i.e. $\nabla\mathbf{m} \approx 0$, is inversely proportional to the damping α_{el} (in the case α_{pd}=0), ${\rm v} \approx (Pj\mu_B/e)((1+\xi^2)/(\xi m_0))$. Then, ignoring the renormalization of the diffusion coefficient D_0, the velocity is expected to take a similar form as in the case which does not account for the diffusion but with ξ replaced by $\xi^N = \tau_{ex}^N/\tau_{sf}^N$. The damping α will be also affected by this renormalization as is expected, since broadening due to inhomogeneities is well known to occur in ferromagnetic resonance measurements.

Now, we turn to the discussion of the results of the above theory for a 1-D DW configuration. We solve numerically the coupled equations of motion for the conduction electrons and that of the magnetization. We include the d-d exchange between the local moments, the anisotropy along the direction of the current and the dipole field. Pinning is neglected but can be easily included in the simulations. Besides varying the width of the DW, we also vary the other parameters in the sd model since there is no universal agreement on their exact values. For example, it is generally believed that spin relaxation times are about two orders of magnitude longer than momentum relaxation times. While this may be true in paramagnets , we already know that in $Ni_{80}Fe_{20}$ they are comparable [24]. In Permalloy, the spin diffusion length, $l_s = {\rm v_f}\sqrt{\tau_{\rm sf}\tau_{\rm p}}$, is of the order of 5 nm which is of the same order as the mean free path, $l_p = 3$ nm.

Figure 1 shows the effect of introducing the (unnormalized) diffusion term D_0 in the equations of motion of the magnetization. For DW width larger than 100 nm our result approximately recovers that of Ref. [9]. The variations of the domain wall velocity v with λ are found to depend strongly on D_0. This is expected since v is, to first order, a function of D_0/λ^2 (cf. inset). Moreover, the velocity peaks when the mean free path of the conduction electrons, l_p, is of the same order as the DW width, since for $l_p \gg \lambda$ there is almost no scattering while for $l_p \ll \lambda$ there is only

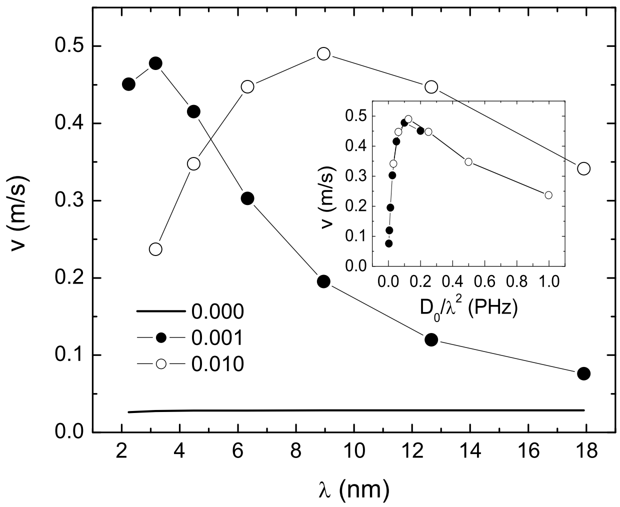

FIG. 1: Domain wall velocity as a function of domain wall width for $j_c = 10^8$ A/cm^2, $\tau_{sf} = 3.0 \times 10^{-12}$ s, $\alpha_{\rm pd} = 0.01$ and three different diffusion coefficients, $\mathcal{D} = D_0\mathbf{I}$, given in the figure in units of m^2/s. A value of $D_0 = 10^{-2}$ m^2/s corresponds to $\tau_p \approx 10^{-14}$ s. The solid thick line is that of Zhang and Li [9]. The inset shows the DW velocity versus $\frac{D_0}{\lambda^2}$.

slow diffusion. We have confined our results to $\lambda \geq 2$ nm since at much smaller DW widths, we expect contributions from Coulomb interactions and a breaking of the quasiclassical picture employed here.

In figure 2, we show the effect of the corrections introduced by the off-diagonal terms in the diffusion tensor. This non-adiabatic effect actually appears to suppress the DW velocity or the effect of diffusion as we explained earlier. Otherwise, the functional behavior of the velocity remains similar to the one discussed in figure 1.

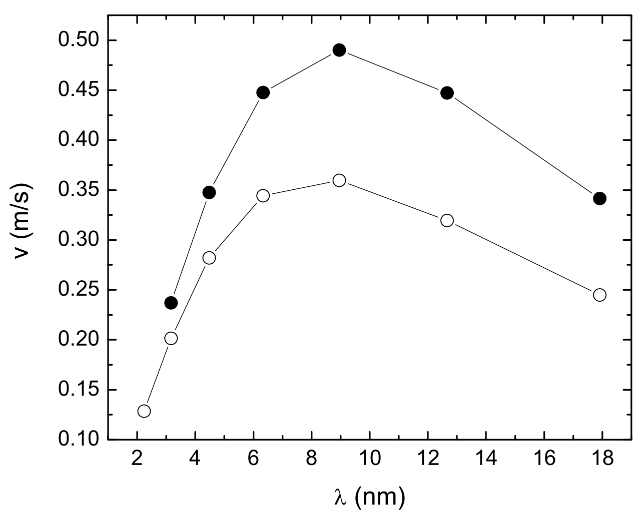

FIG. 2: Domain wall velocity as a function of domain wall width with the correct diffusion tensor taken into account. The solid (open) symbols are without (with) off-diagonal corrections of the diffusion tensor. Parameters are identical to those in Fig. 1

Finally in figure 3, we extract the contribution of the conduction electrons to the effective damping of the magnetization. First, we observe that the off-diagonal diffusion terms have little effect on the relaxation of $\mathbf{M}$ which is mainly determined by the spin relaxation time τ_{sf}. These results are also not sensitive to the DW width and the extracted

electronic damping has the correct order of magnitude for metals.

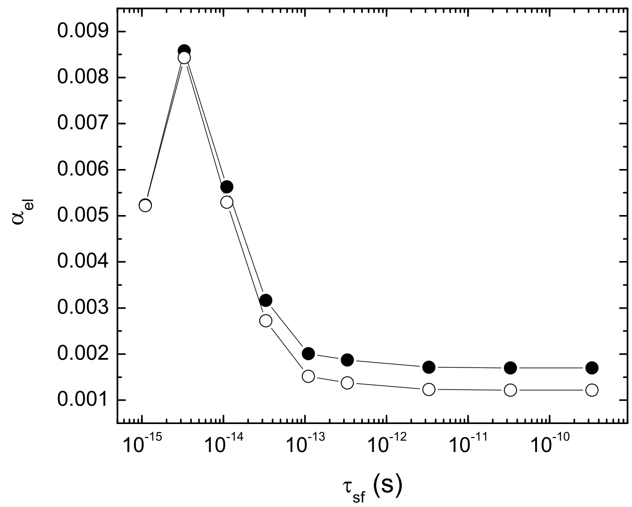

FIG. 3: The electronic damping α_{el} as a function of spin flip scattering τ_{sf} for a 10 nm domain wall. The solid (open) symbols are for off-diagonal terms included (not included). The diffusion constant is $D_0 = 10^{-2}m^2/s$ and $\alpha_{\rm pd} = 0$.

In summary, we have solved the conduction electron-magnetization problem in the presence of a current self-consistently. We found that the diffusion term provides a larger contribution to the drive torque than to the damping process, leading to a overall increase of domain wall velocity. We also showed that the new off-diagonal terms of the diffusion tensor enhance the DW velocities which become at least one order of magnitude larger than previously found. Moreover, the dependence of the DW velocity on the width of the DW was found to be non-linear and strongly dependent on the non-adiabatic behavior of the conduction electrons through the non-diagonal corrections of the diffusion tensor. We have been also able to determine the contribution of the conduction electrons to the damping in ferromagnetic metals which we found to be of the same order as the typical measured values of α. Therefore, our treatment allows us to include electronic damping in micromagnetic calculations in a more rigorous way than is currently done by simply accounting for it by a simple α parameter.

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* Electronic address: mourad.benakli@seagate.com
† Electronic address: julius.hohlfeld@seagate.com
‡ Electronic address: arebei@mailaps.org

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